9/28/16 "Its always too early to quit." -Norman Peale

HW: Who gets voted off on Survivor?
Test 2 on Wednesday 10/19

AIM: What are some basic rules for Differentiation?

Warm Up:

Complete the Mini Quiz

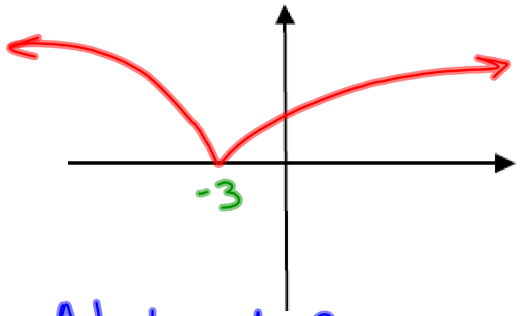
Differentiability Implies Continuity:

If f is differentiable at $x = c$, then f is continuous at $x = c$.

1. If a function is differentiable at $x = c$, then it is continuous at $x = c$. So, differentiability implies continuity.
2. It is possible for a function to be continuous at $x = c$ and not be differentiable at $x = c$. Continuity does not imply differentiability.

$f(x) = (x+3)^{\frac{2}{3}}$

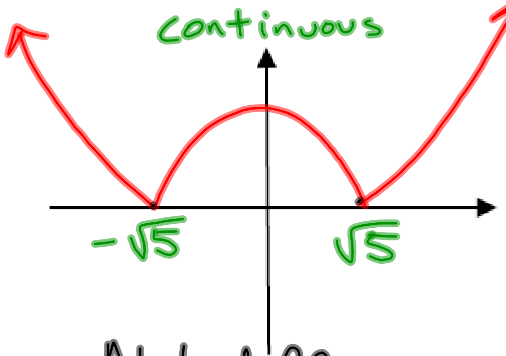
continuous



Not differentiable @ $x = -3$ (cusp)

$g(x) = |x^2 - 5|$

continuous



Not differentiable @ $x = -\sqrt{5}$ or $x = \sqrt{5}$
cusp cusp

* Not differentiable wherever there is a cusp, corner, or vertical tangent line.

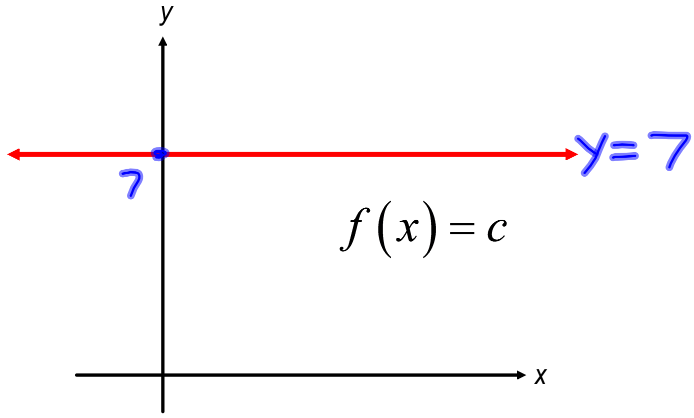
Basic Differentiation Rules

To find derivatives

1. The Constant Rule

The derivative of a constant function is 0.
For any real number, *c* : $\frac{d}{dx}[c]=0$

The slope of a horizontal line is 0.



The derivative of a constant function is 0.

A.) $y=-4$

$y'=0$

B.) $s(t)=45$

$s'(t)=0$

C.) $f(x)=0$

$f'(x)=0$

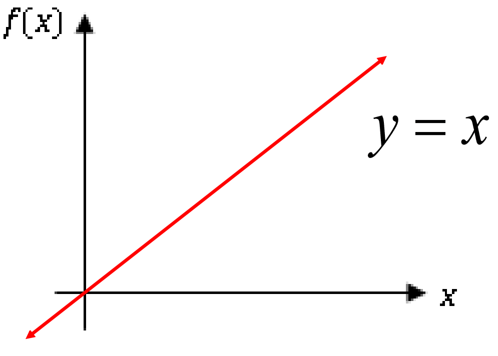
D.) $y=\frac{k\pi}{2}$

$\frac{dy}{dx}$ or $y'=0$

2. The Single Variable Rule:

The derivative of *x* is 1. $\frac{d}{dx}[x]=1$

Think graphically about the line *y* = x.



A.) $f(x)=x$

$f'(x)=1$

B.) $s(t)=t$

$s'(t)=1$

C.) $x(t)=t$

$x'(t)=1$

$f(x) = 2x^{100} + x^{98}$

$f'(x) = 200x^{99} + 98x^{97}$

3. The Power Rule

If n is a rational number then the function x^n is differentiable and $\frac{d}{dx}\left[x^n\right] = n \bullet x^{n-1}$

EX #3: Use the power rule to find the derivatives of the following, if:

A.) $y = x^2$

$y' = 2x^{2-1}$

$\boxed{y' = 2x}$

B.) $f(x) = x^4$

$\boxed{f'(x) = 4x^3}$

C.) $y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{x^{-\frac{1}{2}}}{2} = \frac{1}{2x^{\frac{1}{2}}}$

$y' = \frac{1}{2\sqrt{x}}$

D.) $g(x) = \frac{1}{x} = x^{-1}$

$g'(x) = -1x^{-2}$

$\boxed{g'(x) = -\frac{1}{x^2}}$

E.) $f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-3}$

$\boxed{f'(x) = \frac{-2}{x^3}}$

F.) $y = \frac{1}{x^{2/3}}$

$y = x^{-\frac{2}{3}}$

$y' = -\frac{2}{3}x^{-\frac{5}{3}}$

$y' = \frac{-2}{3x^{\frac{5}{3}}}$

$\boxed{y' = \frac{-2}{3\sqrt[3]{x^5}}}$

4. The Constant Multiple Rule:

The derivative of the term ax^n, where *a* and *n* are real numbers, is $(a \bullet n)x^{n-1}$

STEPS:

1. Multiply the coefficient by the variable's exponent. If no coefficient is stated -- in other words, the coefficient equals 1-- the exponent becomes the new coefficient.
2. Subtract 1 from the exponent.

EX #4: Use the constant multiple rule to find the derivatives of the following, if:

A.) $y = 3x^2$

$y' = 3 \cdot 2x^{2-1}$

$\boxed{y' = 6x}$

B.) $s(t) = -5t^3$

$\boxed{s'(t) = -15t^2}$

C.) $y = 6\sqrt{x} = 6x^{1/2}$

$y' = 3x^{-1/2}$

$y' = \frac{3}{x^{1/2}} = \frac{3}{\sqrt{x}}$

$\boxed{y' = \frac{3\sqrt{x}}{x}}$

D.) $g(x) = \frac{3}{x^2}$

$g(x) = 3x^{-2}$

$g'(x) = -6x^{-3}$

$\boxed{g'(x) = -\frac{6}{x^3}}$

E.) $f(x) = \frac{4}{\sqrt{x}} = \frac{4}{x^{1/2}}$

$f(x) = 4x^{-1/2}$

$f'(x) = -2x^{-3/2}$

$f'(x) = \frac{-2}{x^{3/2}}$

$\boxed{f'(x) = \frac{-2}{\sqrt{x^3}}}$

F.) $y = \frac{-8}{\sqrt[3]{x^4}}$

$y = -8x^{-4/3}$

$y' = \frac{32}{3}x^{-7/3}$

$y' = \frac{32}{3x^{7/3}} = \boxed{\frac{32}{3\sqrt[3]{x^7}}}$

5. The Sum and Difference Rules:

The derivative of a sum or difference is the sum or difference of the derivatives.

Take the Derivative of each piece.

$$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$$

and

$$\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$$

A.) $f(x)=x^3-5x^2+2x-7$ ← constant

$f'(x)=3x^2-10x+2+0$

$f'(x)=3x^2-10x+2$

B.) $y=\frac{2}{5}x^5+x^4+3x^2-1$

$\cancel{5}\cdot\frac{2}{\cancel{5}}$

$\boxed{y'=2x^4+4x^3+6x}$

C.) $g(x)=\sqrt[3]{x}+5\sqrt{x}$

$g(x)=x^{1/3}+5x^{1/2}$

$g'(x)=\frac{1}{3}x^{-\frac{2}{3}}+\frac{5}{2}x^{-1/2}$

$g'(x)=\frac{1}{3x^{2/3}}+\frac{5}{2x^{1/2}}$

$\boxed{g'(x)=\frac{1}{3\sqrt[3]{x^2}}+\frac{5}{2\sqrt{x}}}$

D.) $y=5x^{3/2}-6x^{5/3}+\frac{4}{x}$

$y=5x^{3/2}-6x^{5/3}+4x^{-1}$

$y'=\frac{15}{2}x^{1/2}-\frac{30}{3}x^{\frac{2}{3}}-4x^{-2}$

$\boxed{y'=\frac{15}{2}\sqrt{x}-10\sqrt[3]{x^2}-\frac{4}{x^2}}$

E.) $h(x)=3x^2-\dfrac{4}{x^2}+2\sqrt[2]{x^1}$

$h(x)=3x^2-4x^{-2}+2x^{1/2}$

$h'(x)=6x+8x^{-3}+x^{-1/2}$

$h'(x)=6x+\dfrac{8}{x^3}+\dfrac{1}{x^{1/2}}$

$\boxed{h'(x)=6x+\dfrac{8}{x^3}+\dfrac{1}{\sqrt{x}}}$

F.) $f(x)=\dfrac{8x^3+4x^2-3}{x}=\dfrac{8x^3}{x}+\dfrac{4x^2}{x}-\dfrac{3}{x}$

$f(x)=8x^2+4x-3x^{-1}$

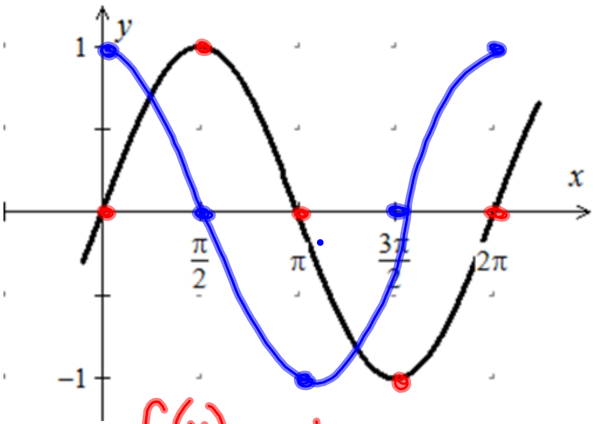
$f'(x)=16x+4+3x^{-2}$

$f'(x)=16x+4+\dfrac{3}{x^2}$

6. Derivatives of Sine and Cosine Functions:

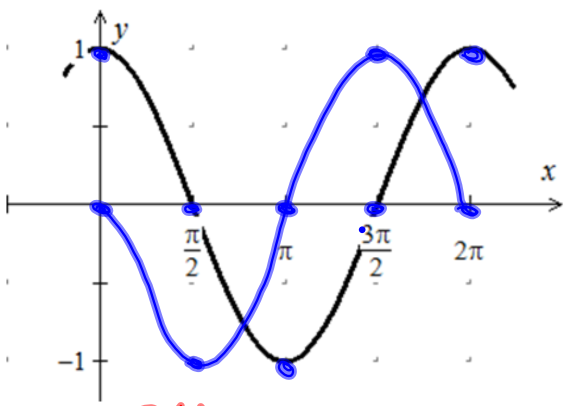
$$\frac{d}{dx}[\sin x] = \cos x \quad \text{and} \quad \frac{d}{dx}[\cos x] = -\sin x$$

EX #4: The derivative of the sine function is the cosine function.



$f(x) = \sin x$

$f'(x) = \cos x$



$f(x) = \cos x$

$f'(x) = -\sin x$

EX #5: Find the derivative of the sine function by using the limit process.

1) $f(x) = x^2 - 2$ ← constant

$\boxed{f'(x) = 2x}$

6) $f(x) = \frac{4}{x^2} - \frac{x^2}{4}$

$f(x) = 4x^{-2} - \frac{1}{4}x^2$

$f'(x) = -8x^{-3} - \frac{1}{2}x$

$\boxed{f'(x) = -\frac{8}{x^3} - \frac{x}{2}}$

3) $f(x) = x^2 + 3x - 6$

$\boxed{f'(x) = 2x + 3}$

2) $f(x) = x - x^3$

$\boxed{f'(x) = 1 - 3x^2}$

9) $f(x) = x\sqrt{3}$

$f(x) = (\sqrt{3})x$

$\boxed{f'(x) = \sqrt{3}}$

8) $f(x) = 3\sqrt{x}$

$f(x) = 3x^{1/2}$

$f'(x) = \frac{3}{2}x^{-1/2}$

$f'(x) = \frac{3}{2x^{1/2}} = \boxed{\frac{3}{2\sqrt{x}}}$

26) $f(x) = \sqrt{3x}$

$f(x) = \sqrt{3}\sqrt{x}$

$f(x) = \sqrt{3}\,x^{1/2}$

$f'(x) = \frac{\sqrt{3}}{2}x^{-1/2}$

$f'(x) = \frac{\sqrt{3}}{2x^{1/2}} = \boxed{\frac{\sqrt{3}}{2\sqrt{x}}}$

25) $f(x) = \frac{x^2}{\ln 2}$

$f(x) = \frac{1}{\ln 2}x^2$ $\qquad f(x) = 3x^2$

$f'(x) = \frac{2}{\ln 2}x = \boxed{\frac{2x}{\ln 2}}$

28) $f(x) = \frac{x^2-1}{x}$

$f(x) = \frac{x^2}{x} - \frac{1}{x}$

$f(x) = x - x^{-1}$

$f'(x) = 1 + x^{-2}$

$\boxed{f'(x) = 1 + \frac{1}{x^2}}$

30) $f(x) = \frac{7x + 3x^2}{5\sqrt{x}}$

$f(x) = \frac{7x}{5\sqrt{x}} + \frac{3x^2}{5\sqrt{x}}$

$f(x) = \frac{7x}{5x^{1/2}} + \frac{3x^2}{5x^{1/2}}$

$f(x) = \frac{7x^{1/2}}{5} + \frac{3x^{3/2}}{5}$

$\frac{1}{2} \cdot \frac{7}{5} = \frac{7}{10}$

$f'(x) = \frac{7}{10}x^{-1/2} + \frac{9}{10}x^{1/2}$

$\boxed{f'(x) = \frac{7}{10\sqrt{x}} + \frac{9\sqrt{x}}{10}}$

10) $f(x) = \frac{x^4}{4} + x - 2$

$f'(x) = \frac{4x^3}{4} + 1$

$\boxed{f'(x) = x^3 + 1}$

12) $f(x) = x^2 - e^2 \leftarrow$ constant

$\boxed{f'(x) = 2x}$